Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. The function f is defined by

$$f(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x+4} \qquad x > 0$$

(a) Show that $f(x) = \dfrac{5x}{x+3}$ **(3)**

(b) Find f^{-1} **(3)**

(c) (i) Find, in simplest form, $f'(x)$.

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer. **(3)**

1. a) $f(x) = \dfrac{5x}{x^2+7x+12} + \dfrac{5x}{x+4} \qquad x>0$

$= \dfrac{5x}{(x+3)(x+4)} + \dfrac{5x}{x+4}$

$= \dfrac{5x + 5x(x+3)}{(x+3)(x+4)}$

$= \dfrac{5x(\cancel{x+4})}{(x+3)(\cancel{x+4})} = \dfrac{5x}{x+3}$

b) to find $f^{-1}(x)$: $\quad f(x) = \dfrac{5x}{x+3}$

① write the function using a "y" and set equal to "x" : $\quad x = \dfrac{5y}{y+3}$

② rearrange to make y the subject : $\quad xy + 3x = 5y$

$y = \dfrac{3x}{5-x}$

③ replace y with $f^{-1}(x)$:

$\therefore f^{-1}(x) = \dfrac{3x}{5-x}$

Question 1 continued

Because we are told to find $f^{-1}(x)$, we must also state the domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

$\therefore$ domain of $f^{-1}(x)$ = range of $f(x)$

$f(x) = \frac{5x}{x+3} \quad x > 0$

→ when $x = 0 \rightarrow f(x) = 0$

→ as $x \rightarrow \infty \rightarrow f(x) = \frac{5}{1 + \frac{3}{x}}$, $\frac{3}{x} \rightarrow 0$

$f(x) \rightarrow 5$

$\therefore$ range of $f(x)$ is $0 < f(x) < 5$

$\therefore f^{-1}(x) = \frac{3x}{5-x} \quad 0 < x < 5$

c) (i) $f(x) = \frac{5x}{x+3}$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 5x \quad \frac{du}{dx} = 5$ | $v = x+3 \quad \frac{dv}{dx} = 1$

$f'(x) = \frac{(x+3)(5) - (5x)(1)}{(x+3)^2} = \frac{\cancel{5x} + 15 - \cancel{5x}}{(x+3)^2}$

$= \frac{15}{(x+3)^2}$

Question 1 continued

(ii) because $f'(x) = \frac{15}{(x+3)^2}$

$(A)^2$ will always be positive for real A

$\therefore f'(x) > 0$ for all $x > 0$

so f is an increasing function

2.

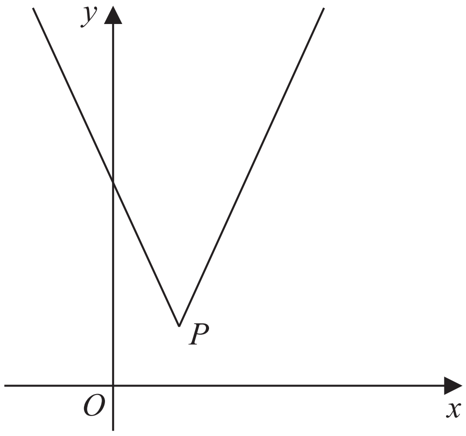


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |3x - 13| + 5 \qquad x \in \mathbb{R}$$

The vertex of the graph is at point P, as shown in Figure 1.

(a) State the coordinates of P.

(2)

(b) (i) State the range of f.

(ii) Find the value of ff(4)

(2)

(c) Solve, using algebra and showing your working,

$$16 - 2x > |3x - 13| + 5$$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x + b)$

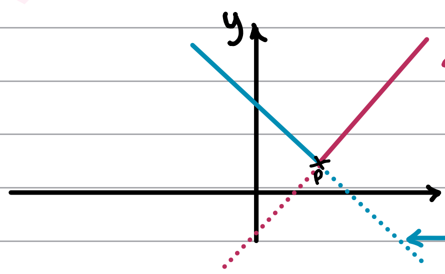
The vertex of the graph with equation $y = a\mathrm{f}(x + b)$ is (4, 20)

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

2.a) $f(x) = |3x - 13| + 5 \qquad x \in R$



$\leftarrow y = (3x - 13) + 5 = 3x - 8$

$\leftarrow y = -(3x - 13) + 5 = -3x + 18$

Question 2 continued

P is the point at which the 2 lines meet

$$3x - 8 = -3x + 18$$

$$6x = 26$$

$$x = \frac{13}{3} \qquad \therefore P = \left(\frac{13}{3}, 5\right)$$

b) (i) range of f(x) as seen from the graph

$$f(x) \geq 5$$

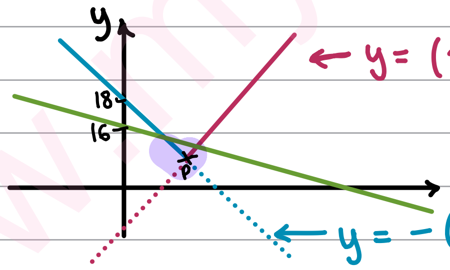
(ii) $f(f(4))$

$$= f(|13(4) - 13| + 5) = f(6)$$

$$= (|13(6) - 13| + 5)$$

$$= 10$$

c) $16 - 2x > |3x - 13| + 5$



The purple section shows where $16 - 2x > f(x)$

$$16 - 2x = -3x + 18 \qquad\qquad 16 - 2x = 3x - 8$$

$$x = 2 \qquad\qquad 5x = 24$$

$$x = \frac{24}{5}$$

$$\therefore \quad 2 < x < \frac{24}{5}$$

Question 2 continued

d) $y = af(x+b)$

stretch
scale factor a
parallel to y-axis

translation
through the vector
$\begin{pmatrix} -b \\ 0 \end{pmatrix}$

$\therefore \quad P = \left(\frac{13}{3}, 5\right) \rightarrow \left(\frac{13}{3} - b, 5a\right) = (4, 20)$

$a = 4$

$b = \frac{1}{3}$

P 6 9 2 0 4 A 0 8 3 2

3.

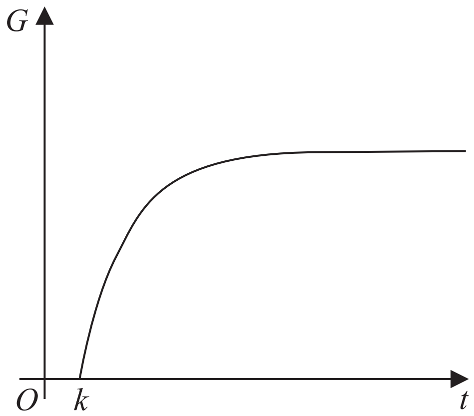


Figure 2

The total mass of gold, G tonnes, extracted from a mine is modelled by the equation

$$G = 40 - 30e^{1-0.05t} \qquad t \geqslant k \qquad G \geqslant 0$$

where t is the number of years after 1st January 1800.

Figure 2 shows a sketch of G against t.

Use the equation of the model to answer parts (a), (b) and (c).

(a) (i) Find the value of k.

(ii) Hence find the year and month in which gold started being extracted from the mine.

(3)

(b) Find the total mass of gold extracted from the mine up to 1st January 1870.

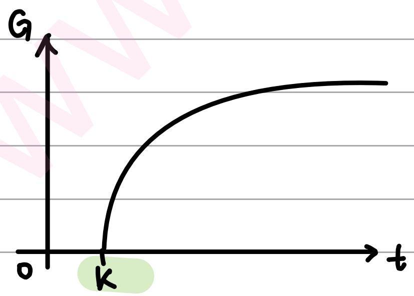
(2)

There is a limit to the mass of gold that can be extracted from the mine.

(c) State the value of this limit.

(1)

3. a) (i) $G = 40 - 30e^{1-0.05t}$ $\quad t \geq k \quad G \geq 0$



when $y = 0 \rightarrow x = k$

$0 = 40 - 30e^{1-0.05k}$

$e^{1-0.05k} = \frac{4}{3}$

$1 - 0.05k = \ln\left(\frac{4}{3}\right) \rightarrow k = 14.2$

P 6 9 2 0 4 A 0 1 0 3 2

Question 3 continued

(ii) 14.2 yrs = 14 yrs 3 months

Jan 1800 + 14yrs 3 months

= April 1814

b) 1st Jan 1800 $\longrightarrow$ 1st Jan 1870

70 yrs

$\therefore G = 40 - 30e^{1-0.05(70)}$

$= 40 - 30e^{-2.5} = 37.5$ tonnes

c) as $t \to \infty$ $e^{1-0.05t} \to 0$

$\therefore G = 40 - \cancel{30e^{1-0.05t}}$ 0

$= 40$ tonnes

Q3

(Total 6 marks)

4. **In this question you should show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\sin(\theta - 30°) = 5\cos\theta$$

can be written in the form

$$\tan\theta = 2\sqrt{3}$$

(4)

(b) Hence, or otherwise, solve for $0 \leqslant x \leqslant 360°$

$$2\sin(x - 10°) = 5\cos(x + 20°)$$

giving your answers to one decimal place.

(3)

4. a) $2\sin(\theta - 30) = 5\cos(\theta)$

USING COMPOUND ANGLE FORMULAE $\rightarrow$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$2(\sin(\theta)\cos(30) - \cos(\theta)\sin(30)) = 5\cos(\theta)$

$\sqrt{3}\sin(\theta) - \cos(\theta) = 5\cos(\theta)$

$\sqrt{3}\sin(\theta) = 6\cos(\theta)$

$\tan(\theta) = 2\sqrt{3}$

b) $2\sin(x-10) = 5\cos(x+20)$

$x - 10 = \theta - 30$

$x = \theta - 20$ $\qquad$ $\theta = \arctan(2\sqrt{3})$

$x = 53.9°$ $\cup$ $233.9°$

P 6 9 2 0 4 A 0 1 2 3 2

5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,dx$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7 dx$$

(2)

5. (i) $\int_2^4 \frac{8}{(2x-3)^3}\,dx$

$u = 2x-3 \qquad \frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

$\therefore \int_1^5 \frac{8}{u^3} \times \frac{du}{2}$

$= \int_1^5 4u^{-3}\,du$

$= \left[-\frac{4u^{-2}}{2}\right]_1^5 = -\frac{2}{5^2} + \frac{2}{1^2} = \frac{48}{25}$

(ii) $\int x\,(x^2+3)^7\,dx$

$u = x^2+3 \qquad \frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$

$\int \cancel{x}\,u^7 \times \frac{du}{2\cancel{x}}$

$= \int \frac{u^7}{2}\,du$

If no limits are given, don't forget constant of integration

$= \frac{u^8}{16} + c \quad = \frac{(x^2+8)^8}{16} + c$

6. (i) The curve C_1 has equation

$$y = 3\ln(x^2 - 5) - 4x^2 + 15 \qquad x > \sqrt{5}$$

Show that C_1 has a stationary point at $x = \frac{\sqrt{p}}{2}$ where p is a constant to be found.
(4)

(ii) A different curve C_2 has equation

$$y = 4x - 12\sin^2 x$$

(a) Show that, for this curve,

$$\frac{dy}{dx} = A + B\sin 2x$$

where A and B are constants to be found.

(b) Hence, state the maximum gradient of this curve.
(4)

6.(i) $y = 3\ln(x^2 - 5) - 4x^2 + 15 \qquad x > \sqrt{5}$

stationary point when $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{3 \times (2x)}{x^2 - 5} - 8x$$

$$\frac{6x}{x^2-5} - 8x = 0$$

$$6x = 8x(x^2 - 5)$$

$$6x = 8x^3 - 40x$$

$$8x^3 = 46x$$

$$x^2 = \frac{23}{4} \qquad x = \frac{\sqrt{23}}{2} \qquad p = 23$$

Question 6 continued

(ii) a) $C_2: \quad y = 4x - 12\sin^2(x)$

$$\frac{dy}{dx} = 4 - 12(2)(\cos(x))(\sin(x))$$

$$= 4 - 24\cos(x)\sin(x)$$

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

$$\therefore \quad \sin(2x) = 2\sin(x)\cos(x)$$

$$\frac{dy}{dx} = 4 - 12(2\sin(x)\cos(x))$$

$$= 4 - 12\sin(2x)$$

$A = 4$
$B = -12$

b) gradient of $C_2 = \frac{dy}{dx}$

max value of $\frac{dy}{dx} = \max(4 - 12\sin(2x))$

$$-1 \leq \sin(2x) \leq 1$$

$$-12 \leq -12\sin(2x) \leq 12$$

$$\therefore \max\left(\frac{dy}{dx}\right) = 4 + 12 = 16$$

Q6

(Total 8 marks)

7 The mass, M kg, of a species of tree can be modelled by the equation

$$\log_{10}M = 1.93\log_{10}r + 0.684$$

where r cm is the base radius of the tree.

The base radius of a particular tree of this species is 45 cm.

According to the model,

(a) find the mass of this tree, giving your answer to 2 significant figures.
(2)

(b) Show that the equation of the model can be written in the form

$$M = pr^q$$

giving the values of the constants p and q to 3 significant figures.
(3)

(c) With reference to the model, interpret the value of the constant p.
(1)

7. a) $\log_{10} M = 1.93\log_{10} r + 0.684$

$r = 45$ cm

$\log_{10} M = 1.93\log_{10}(45) + 0.684 = 3.8747$

LOG RULES → $\log_a b = c \rightarrow a^c = b$

$M = 10^{3.8747} = 7500$ kg

b) $\log_{10} M = 1.93\log_{10} r + 0.684$

$\log_{10} M = \log_{10} r^{1.93} + 0.684$

$\log_{10} M - \log_{10} r^{1.93} = 0.684$ $\quad \log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$

$\log_{10}\left(\frac{M}{r^{1.93}}\right) = 0.684$

Question 7 continued

$$\frac{M}{r^{1.93}} = 10^{0.684}$$

$$M = 10^{0.684} r^{1.93}$$

$p = 10^{0.684}$ $\quad q = 1.93$

c) Because $M = pr^q$

when $r = 1$

$\hookrightarrow M = p(1)^q = p$

$\therefore$ p is the mass of a tree with radius 1cm

Q7

(Total 6 marks)

8. A curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \arcsin\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2 \qquad -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$$

(a) Sketch C.

(1)

(b) Given $x = 2\sin y$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{A - x^2}}$$

where A is a constant to be found.

(3)

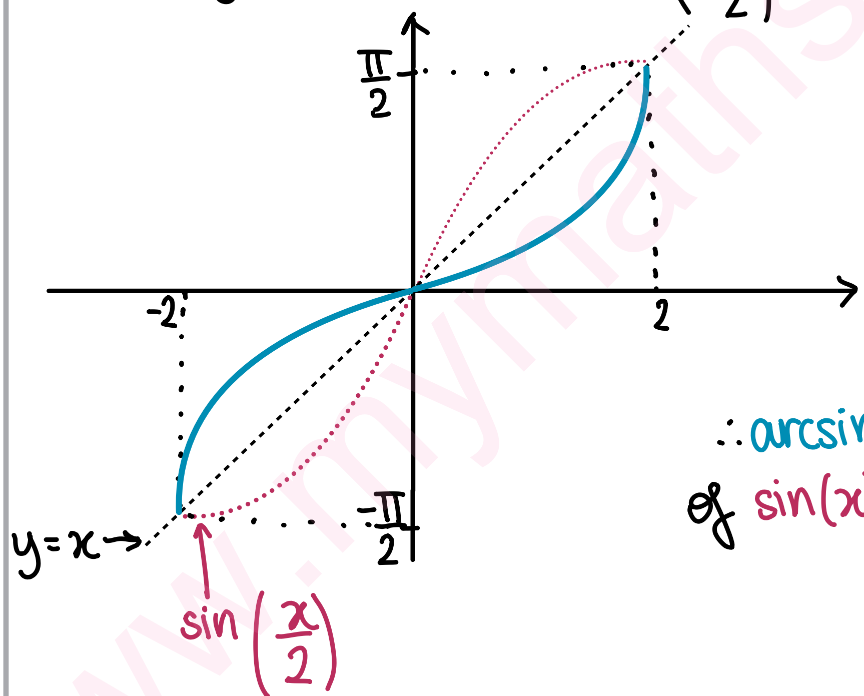
The point P lies on C and has y coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

8.a) $y = f(x) = \arcsin\left(\frac{x}{2}\right)$

$-2 \leq x \leq 2$
$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



$\arcsin(x)$ is the inverse function of $\sin(x)$

$\therefore \arcsin(x)$ is the reflection of $\sin(x)$ in the line $y = x$

P 6 9 2 0 4 A 0 2 2 3 2

Question 8 continued

b) $x = 2\sin(y)$

$\frac{dx}{dy} = 2\cos(y)$

$\boxed{\frac{dy}{dx} = \frac{1}{(dx/dy)}}$

$\therefore \frac{dy}{dx} = \frac{1}{2\cos(y)}$

Given that $x = 2\sin(y)$

$\sin(y) = \frac{x}{2}$

$\cos(y) = \sqrt{1 - \left(\frac{x}{2}\right)^2}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1 - \frac{x^2}{4}}} = \frac{1}{\sqrt{4 - x^2}}$ $A = 4$

c) gradient of tangent to C at P $= \frac{dy}{dx}$ at P

$P \rightarrow \left(x, \frac{\pi}{4}\right)$

$x = 2\sin\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$

so $P = \left(\sqrt{2}, \frac{\pi}{4}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{4 - (\sqrt{2})^2}} = \frac{1}{\sqrt{4 - 2}}$

$= \frac{\sqrt{2}}{2}$

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

gradient

Question 8 continued

Known point on line : $\left(\sqrt{2}, \frac{\pi}{4}\right)$ gradient $= \frac{\sqrt{2}}{2}$

$$y - \frac{\pi}{4} = \frac{\sqrt{2}}{2}\left(x - \sqrt{2}\right)$$

$$y = \frac{\sqrt{2}}{2}x - 1 + \frac{\pi}{4}$$

P 6 9 2 0 4 A 0 2 4 3 2

9.

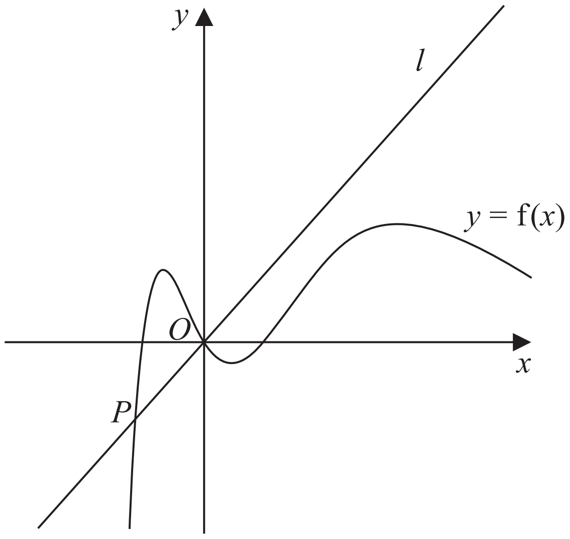


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x(x^2 - 4)\mathrm{e}^{-\frac{1}{2}x}$$

(a) Find $\mathrm{f}'(x)$.

(2)

The line l is the normal to the curve at O and meets the curve again at the point P.

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x}}$$

(4)

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x_n}} \qquad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of x_2

(ii) the x coordinate of P.

(3)

9. a)

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

Question 9 continued

$$f(x) = (x^3 - 4x)\, e^{-\frac{1}{2}x}$$

$u = x^3 - 4x$ $\quad\quad$ $\frac{du}{dx} = 3x^2 - 4$

$v = e^{-\frac{x}{2}}$ $\quad\quad$ $\frac{dv}{dx} = -\frac{1}{2} e^{-\frac{x}{2}}$

$$f'(x) = (3x^2 - 4)(e^{-\frac{x}{2}}) + (x^3 - 4x)\left(-\frac{1}{2} e^{-\frac{x}{2}}\right)$$

$$= e^{-\frac{x}{2}}\left(3x^2 - 4 - \frac{x^3}{2} + 2x\right)$$

b) L is normal to O

$m_L = -\frac{1}{m_{\text{curve at O}}}$

gradient of curve at O

$= \frac{dy}{dx} = e^{-\frac{0}{2}}(-4) = -4$

$\therefore m_L = \frac{-1}{-4} = \frac{1}{4}$

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

gradient

Known point : $(0, 0)$

$$y - 0 = \frac{1}{4}(x - 0) \rightarrow y = \frac{x}{4}$$

Line L meets curve : $\frac{\cancel{x}}{4} = \cancel{x}(x^2 - 4)e^{-\frac{x}{2}}$

$$e^{\frac{x}{2}} = 4x^2 - 16$$

Question 9 continued

$$x^2 = \frac{1}{4}\left(e^{\frac{x}{2}} + 16\right)$$

$$x = \pm\sqrt{\frac{1}{4}\left(e^{\frac{x}{2}}+16\right)}$$

Because P is in the 3rd quadrant, x coordinate of P is negative

$$\therefore\ x = -\frac{1}{2}\sqrt{e^{\frac{x}{2}}+16}$$

c) $x_{n+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{x_n}{2}}}$

$x_1 = -2$

$x_2 = x_{1+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{-2}{2}}} = -\frac{1}{2}\sqrt{16+e^{-1}} = 2.0229$

$x_3 = -2.0226$

$x_4 = -2.0226$

$\therefore$ to 4 d.f. $x = -2.0226$

10.

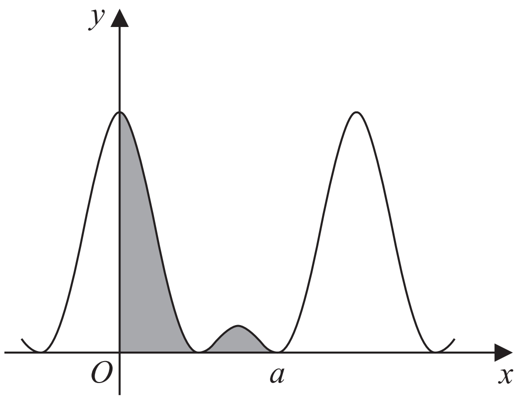


Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where p, q and r are constants to be found.

(2)

The curve touches the positive x-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y-axis and the x-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

10. a) $y = (1 + 2\cos(2x))^2$

$(1 + 2\cos(2x))^2 = 1 + 4\cos(2x) + 4\cos^2(2x)$

$\cos(2A) = \cos^2(A) - \sin^2(A) = \cos^2(A) - (1 - \cos^2(A))$

$= 2\cos^2(A) - 1$

$\therefore \cos(4x) = 2\cos^2(2x) - 1$

$\hookrightarrow 2\cos^2(2x) = \cos(4x) + 1$

$(1 + 2\cos(2x))^2 = 1 + 4\cos(2x) + 2(\cos(4x) + 1)$

Question 10 continued

$$= 3 + 4\cos(2x) + 2\cos(4x)$$

b) Area under curve $= \int_a^b y \, dx$

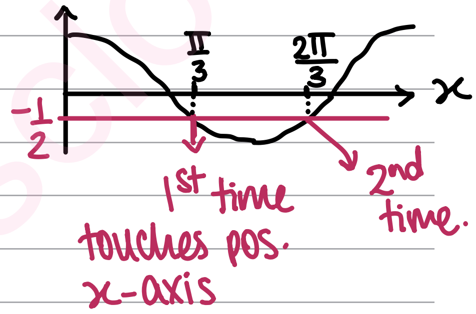
a is when the curve touches positive x-axis for 2^{nd} time

$\therefore y = 0$

$$(1 + 2\cos(2x))^2 = 0$$

$$1 + 2\cos(2x) = 0$$

$$\cos(2x) = -\frac{1}{2}$$



$$\therefore a = \frac{2\pi}{3}$$

Area:
$$\int_0^a (1 + 2\cos(2x))^2 \, dx$$

$$= \int_0^a 3 + 4\cos(2x) + 2\cos(4x) \, dx$$

$$= \left[3x + \frac{4}{2}\sin(2x) + \frac{2}{4}\sin(4x) \right]_0^a$$

$$= 3a + 2\sin(2a) + \frac{\sin(4a)}{2}$$

$$= 3\left(\frac{2\pi}{3}\right) + 2\sin\left(2\left(\frac{2\pi}{3}\right)\right) + \frac{1}{2}\sin\left(4\left(\frac{2\pi}{3}\right)\right)$$

$$= 2\pi - \frac{3\sqrt{3}}{4}$$